The Transistor as a Network Element*

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The development of the transistor has provided an active element having important advantages in space and power. As a result, the question arises whether strategic insertion of such active elements in passive networks might lead to interesting results. This paper gives a theoretical analysis confirmed by experiment, of certain possible network applications of transistors. Four general areas are considered in which transistors are used as follows: to reduce the detrimental effects of dissipative reactive elements, to eliminate the necessity for inductors in frequency selective circuits, to produce two terminal envelope delay structures having zero loss, and to invert the impedance of reactive structures. The conclusion is drawn that judicious interspersion of transistors in a transmission network enables performance to be achieved which would otherwise be unobtainable or uneconomical.

INTRODUCTION

It has become customary through the years to classify linear circuits as either active or passive. This convenient, but arbitrary, division has encouraged a philosophy that regards each as a separate and distinct domain. The recent spectacular advances in active devices suggest that in some cases the traditional boundaries should be erased and that a unified approach should be made.

In particular the development of the transistor offers the possibility of interspersing small active elements throughout a passive network to achieve certain desirable effects. This paper intends to survey a few of the ways in which a transistor can be used to advantage in transmission networks. The discussion is divided into four parts as follows:

1. Reduction of dissipation.
2. Elimination of inductance.
3. Production of delay.
4. Inversion of impedance.

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The first portion offers a new approach to the everpresent problem of imperfect reactive elements. The second portion discusses a method of combining resistance and capacitance with transistors to produce characteristics conventionally realized by inductance and capacitance. The third portion proposes a technique for obtaining any specified delay characteristic with a two terminal active structure. The fourth portion considers means of using a transistor to transform passive elements of ordinary size into passive elements of greatly reduced size.

I. REDUCTION OF DISSIPATION

For simplicity in the treatment of network problems it is frequently assumed that purely reactive elements will be used. In many cases this approximation is satisfactory; other times it is worthless, and a more realistic analysis must be made. In this latter case one possibility is to nullify the unwanted dissipation by means of a bridge balance.[1] This will entail the acceptance of some flat loss. Another possibility is to insert active elements within the network in order to supply just enough energy to offset the inherent dissipation of the elements. This second approach, until now relatively unexplored, is being tried with promising results. To avoid introducing new terminology the discussion will employ the concept of negative resistance which has been studied with interest by many investigators.[2-18]

Negative Resistance

Negative resistance is a misleadingly simple name applied to a complex phenomenon. The term implies behavior in some opposite sense to that of an ordinary positive resistance. This is true only for a limited range of frequencies and signal levels. As generally used negative resistance refers to a two terminal active network or electronic device in which the voltage-current ratio has a negative real part and negligible imaginary part.

TABLE I — NEGATIVE RESISTANCE

Parameter	Shunt Type	Series Type
Independent variable	Voltage controlled	Current controlled
Required external impedance	Short circuit stable	Open circuit stable
Effect of internal gain reduction	Increased magnitude of R_N	Decreased magnitude of R_N
Associated reactance	Parallel capacitance	Series inductance

For convenience the simple forms of negative resistance may be divided into two general classes which are duals in a network sense. Since these classes have been identified in the literature in several different ways, it seems desirable to summarize the major characteristics in the form given in Table I.

Therefore a shunt negative resistance is one whose magnitude is controlled mainly by the voltage across its terminals. It is short circuit stable which means it must operate into a low impedance. When the internal gain used to produce the effect is reduced, the magnitude of a shunt negative resistance increases. In addition it should be associated with a parallel capacitance to predict its behavior outside the working band of frequencies.

One method of producing a two terminal shunt negative resistance is to arrange a transistor as shown in Fig. 1(a). To facilitate prediction of the behavior of this combination it is desirable to derive an equivalent circuit.

Equivalent Circuit of a Transistor Shunt Negative Resistance

An equivalent circuit of the transistor and its associated network is shown in Fig. 1(b). By denoting each condenser reactance as jX, the circuit determinant, Δ, can be written as follows.

$$\begin{vmatrix} j2X & -jX & -jX & 0 \\ -jX & r_b + r_e + R_f + jX & -R_f & -r_e \\ -jX & -R_f & R_f + R_a + jX & -R_a \\ 0 & r_m - r_e & -R_a & r_e + r_c - r_m + R_a \end{vmatrix}$$

Next the input impedance is determined as Δ/Δ_{11}.

From this formula the exact general expression for the input impedance is found to be very cumbersome and will not be given. A useful approximation can be found by making some simplifying assumptions

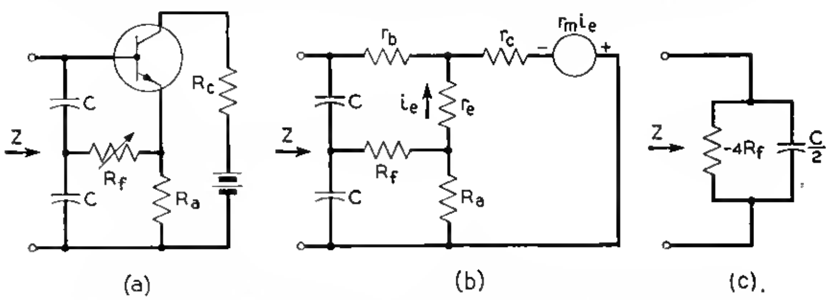


Fig. 1 — Equivalent circuit of transistor shunt negative resistance.

as follows:

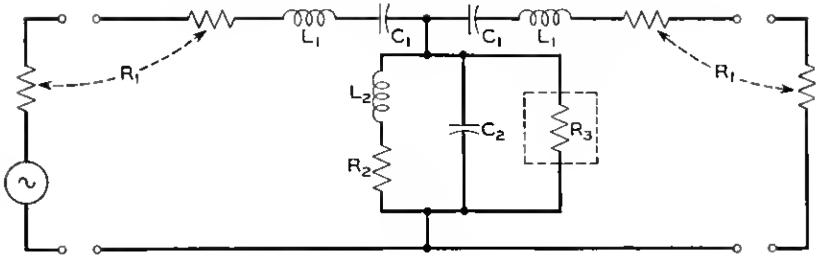
$$\text{Let } r_c >> r_b \qquad r_m >> r_b$$
$$r_c >> r_e + R_a \qquad r_m >> r_e + R_a$$

Under these conditions the network can be represented by the equivalent circuit shown in Fig. 1(e). This circuit consists of a parallel combination of resistance and capacitance in which the capacitance is that of the original two capacitances in series and the resistance is negative and equal to four times the feedback resistor, R_f. Hence the magnitude of the generated shunt negative resistance can be controlled by adjustment of R_f. One measure of the accuracy of this approximation is how much the "constants" of the equivalent circuit change with frequency. Calculations of a typical case show that deviations in frequency of ± 5 per cent cause deviations in both capacitance and negative resistance of ± 0.05 per cent. Hence this approximation is very accurate for narrow band applications.

This circuit can now be used to advantage in a band filter.

Confluent Band Filter

A conventional confluent band filter is shown in Fig. 2(a). In this structure the presence of dissipation in the series branches impairs performance by introducing flat loss, whereas any dissipation in the shunt branch not only produces flat loss, but, worse still, causes rounding of the transmission characteristic at the edges of the band. For narrow filters having a small percentage band width, any appreciable dissiptation in the shunt arm can degrade the transmission characteristic beyond a reasonable tolcrance. One good answer to this problem is to use elements having an extremely low resistive component such as quartz crystals. However, quartz is expensive and has other limitations. Another solution is to build a negative resistance into the filter so as to reduce the inherent element dissipation to zero or at least to a tolerable value. In the present case a shunt negative resistance will be used to compensate the shunt branch. This is done by splitting the shunt capacitance of Fig. 2(a) and inserting the circuit of Fig. 1(a). This can be illustrated by an example. When the filter of Fig. 2(a) is designed to give a 5 per cent band at a midfrequency of 10 kc and impedance level of 600 ohms the shunt branch offers an undesirably low impedance to the compensating transistor circuit and in addition requires cumbersome element values. Both difficulties can be corrected by using capacitative impedance transformations on each side of the shunt branch, thereby



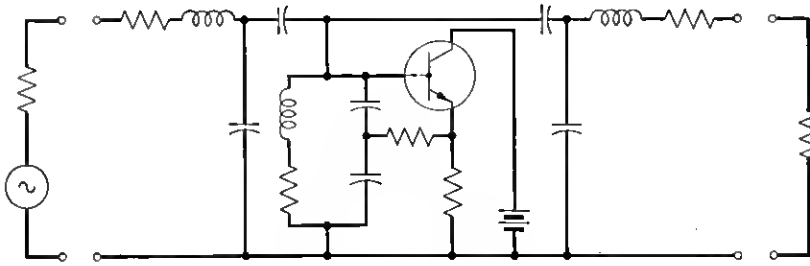


Fig. 2 — Confluent band filters. (a) Conventional. (b) Active.

raising the impedance of the shunt branch without changing the impedance level at the input and output terminals. At the same time the elements assume much more reasonable values. The modified configuration together with the active portion is shown in Fig. 2(b).

Using the filter described above, a series of transmission curves were calculated and are shown in Fig. 3. When ideal elements are assumed, the transmission is

$$e^{-\theta} = \frac{k^3 p^3}{(p^2 + kp + 1)\,[p^4 + kp^3 + (k^2 + 2)\,p^2 + kp + 1]}$$

where $k = \dfrac{f_2 - f_1}{\sqrt{f_2 f_1}}$ and $p = j\omega$

Calculation of this expression results in the classical characteristic labeled "Ideal Passive". When, however, typical values of element resistance are introduced, the transmission is

$$e^{-\theta} = \frac{2R_1\,(\delta_1 p + \delta_2 p^2)\,(\beta_1 p + \beta_2 p^2 + \beta_3 p^3)}{(\alpha_0 + \alpha_1 p + \alpha_2 p^2 + \alpha_3 p^3 + \alpha_4 p^4)^2 - (\delta_1 p + \delta_2 p^2)^2}$$

where

$$
\begin{aligned}
\alpha_0 &= R_1(1 + g) \\
\alpha_1 &= R_1[T_1(1 + g) + T_2 + T_{23}] + R_2T_1 \\
\alpha_2 &= R_1[A_1(1 + g) + T_1(T_2 + T_{23}) + A_2] + R_2T_1T_{22} \\
\alpha_3 &= R_1[A_1(T_2 + T_{23}) + A_2T_1] & g &= \frac{R_2}{R_3} \\
\alpha_4 &= R_1A_1A_2 & T_1 &= R_1C_1 \\
\delta_1 &= R_2T_1 & T_2 &= R_2C_2 \\
\delta_2 &= R_2T_1T_{22} & T_{22} &= \frac{L_2}{R_2} \\
\beta_1 &= T_1(1 + g) & T_{23} &= \frac{L_2}{R_3} \\
\beta_2 &= T_1(T_2 + T_{23}) & A_1 &= L_1C_1 \\
\beta_3 &= T_1A_2 & A_2 &= L_2C_2
\end{aligned}
$$

This expression with the compensating negative resistance $R_3 = \infty$ produces the characteristic labeled "Practical Passive".

The dotted curves show the effect of adding various amounts of negative resistance to the shunt branch by letting R_3 assume negative values. The number on each dotted curve is the ratio of the resistive component of the shunt arm at anti-resonance to the magnitude of the compensating negative resistance. For example, for the curve labeled $\rho = 1$, the resistance in the shunt arm is entirely compensated so that the loss is only that due to the resistance in the series arms. By increasing the amount of compensation in the shunt arm so that $\rho > 1$, called overcompensation it is possible effectively to nullify the losses in the series arm as well. Comparison of the active curve labeled $\rho = 1.21$ with the "ideal passive" curve shows that this technique of resistance compensation can produce a practical filter having a characteristic equal to that of a filter having ideal elements. Filters of this type have already been successfully used in field test equipment.

When the degree of compensation is increased still further, the filter begins to provide gain in the band as shown by the $\rho = 1.36$ curve. It is clear that continued increases in the compensation will eventually absorb the terminations causing the structure to become unstable.

Although the curves given in Fig. 3 are all calculated, tests on experimental models show excellent agreement. To a reader having long experience with passive filters the development of negative insertion loss may seem a little surprising. In order to lend an air of authenticity to this midband gain it is instructive to consider the behavior of a resistive tee section having a negative element. This is a reasonable analogue,

because at midfrequency the reactive component becomes zero in each branch of the filter.

Transmission of Symmetrical Tee

The insertion loss of a symmetrical tee section with positive resistance is a well known concept. It is doubtful, however, if the behavior of a tee with a negative element is equally well known. Consider the section shown in Fig. 4 operating between terminations R and having series arms, R_A and a shunt arm, R_B. Normalize by letting $a = R_A/R$ and $b = R_B/R$. Insertion loss is plotted vs. b with a as the third parameter. For b positive the usual loss pattern results; for b negative, a more complex situation develops. When b is very large and negative, the section is still producing a small loss, but as b becomes smaller in magnitude the loss drops to zero and finally becomes a gain. There is a lower limit on the magnitude of b beyond which oscillations will occur. This limit is reached when $2b = -(a + 1)$.

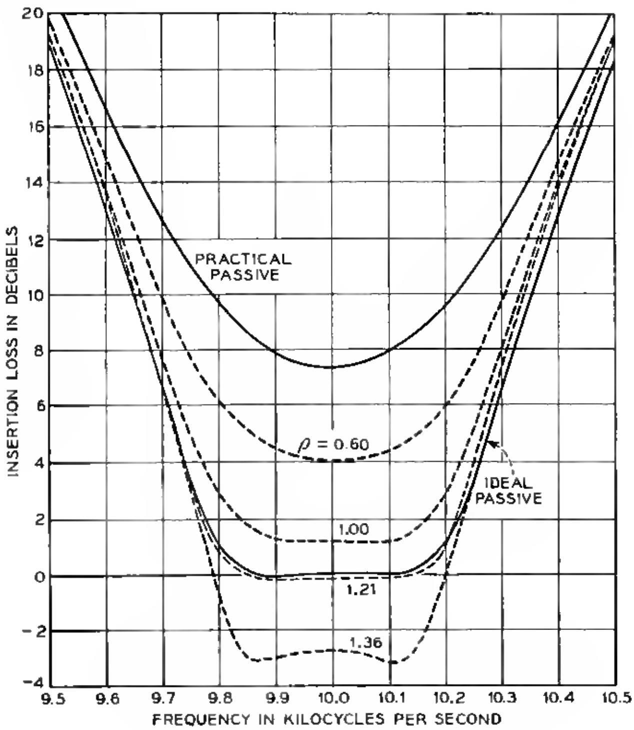


Fig. 3 — Transmission of confluent band filters.

Singularities of Confluent Band Filter

Recent work on insertion loss design and potential analog methods by S. Darlington and others has fostered the practice of characterizing a network by plotting its natural modes and infinite loss points in the complex frequency plane. In the present case it is instructive to study the effect that reducing dissipation will bave on the singularities. A full section confluent band filter has five infinite loss points and eight natural modes. In Fig. 5 the singularities of the passive, confluent band filter discussed earlier are plotted in the complex frequency plane and identified by the digit one. A single infinite loss point or zero lies on the negative sigma axis, a pair falls at the origin, and a conjugate pair is located near the midband frequency. The natural modes or poles consist of two conjugate double pairs situated at about the upper and lower cut off frequencies of the filter. The distance of the complex singularities from

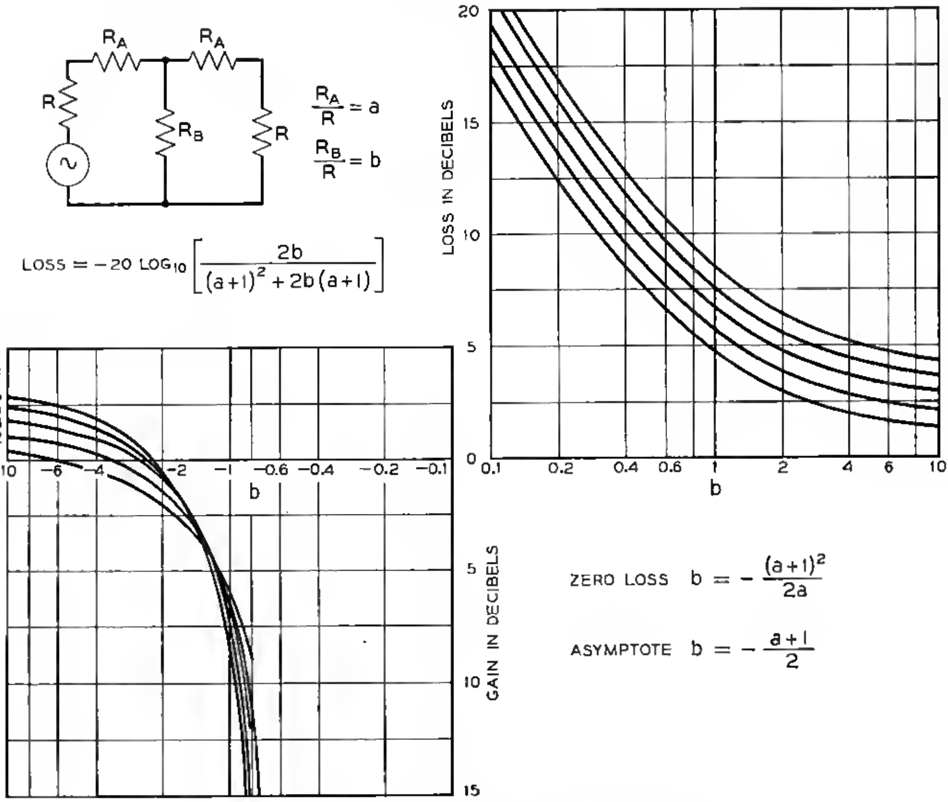


Fig. 4. Transmission of symmetrical tee section.

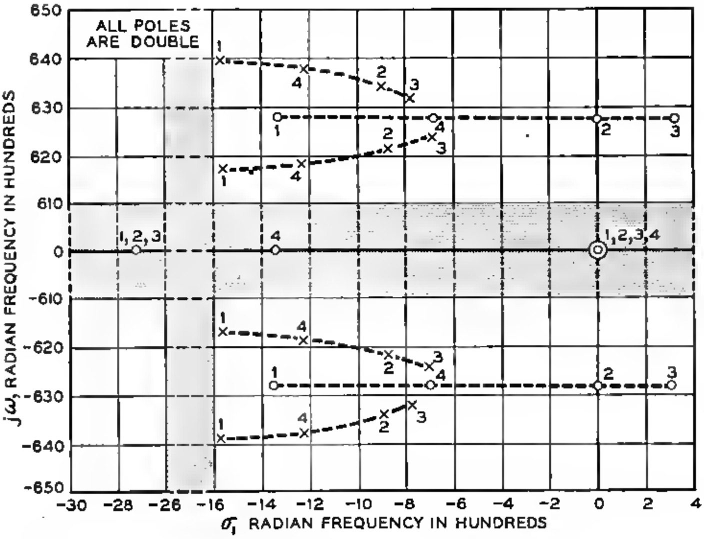


Fig. 5 — Effect of reducing dissipation on singularities of confluent band filter.

the real frequency axis is a function of the amount of dissipation in the elemets. When a value of negative resistance corresponding to the $\rho = 1$ curve in Fig. 3 is added to the passive filter, the singularities move from positions marked 1 to those marked 2 in Fig. 5. Adding a larger amount of negative resistance corresponding to the $\rho = 1.21$ curve in Fig. 3 produces the singularities marked 3. It should be noted that the infinite loss point on the negative sigma axis as well as the two at the origin have not moved. If the dissipation in the shunt branch is reduced by removing the coil and replacing by one having half as much resistance, the singularities change from position one to position four. In this case the infinite loss point on the sigma axis does move. This illustrates that the change in pattern of singularities resulting from use of negative resistance is similar to, but not the same as, that resulting from use of passive inductors having higher values of Q.

M-Derived Band Pass

In order to provide a sharp cut-off in a filter use is often made of m-derived peak sections. In the configuration shown in Fig. 6 loss peaks will occur at selected frequencies above and below the pass band provided the elements are nearly free of dissipation. The closer the attenuation peaks are to the pass band the more nearly free from dissipation the

elements must be for good performance. As in the previous case, a transistor negative resistance is used to compensate the anti-resonant portion of the shunt arm. The magnitude of this resistance can also be adjusted to serve the additional purpose of compensating for resistance in the series resonant circuit in the shunt arm, as well as the series resonant circuits in the series arms.

The transmission of a non-dissipative m-derived band filter between unit resistive terminations is

$$e^{-\theta} = \frac{kp[(1 - m^2)p^4 + (2 - 2m^2 + k^2)p^2 + (1 - m^2)]}{(mp^2 + kp + m)[p_4 + kmp^3 + (k^2 + 2)p^2 + kmp + 1]}$$

where $m = \sqrt{1 - \left(\frac{f_c}{f_\infty}\right)^2}$

Assuming no dissipation a peak section with an m of 0.86 will give the characteristic shown in Fig. 7 labeled "Ideal Passive". However, when this filter is constructed with typical elements the curve labeled "Practical Passive" results. By introducing a suitable amount of negative resistance the transmission of the practical filter can be made comparable to that of the ideal filter, as illustrated by the curve labeled "Practical Active".

For maximum utility active filter sections must be capable of being connected in tandem to form composite filters without instability, reflections, or interactions. Fig. 8 shows that these filters meet this requirement by giving the measured transmission of a band filter composed of two dissimilar peak sections. On the basis of attainable electrical charac-

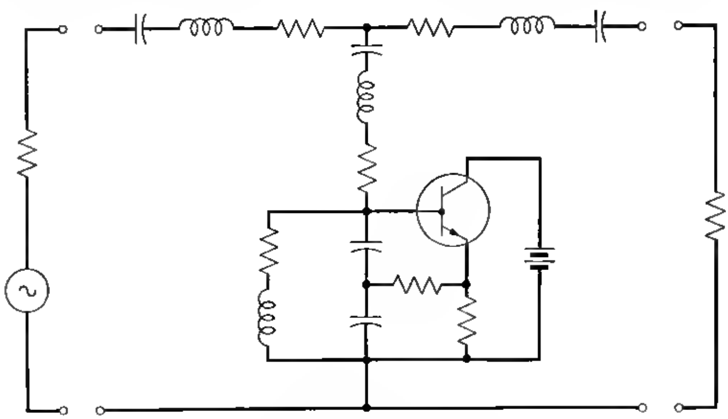


Fig. 6 — Active M-derived band filter.

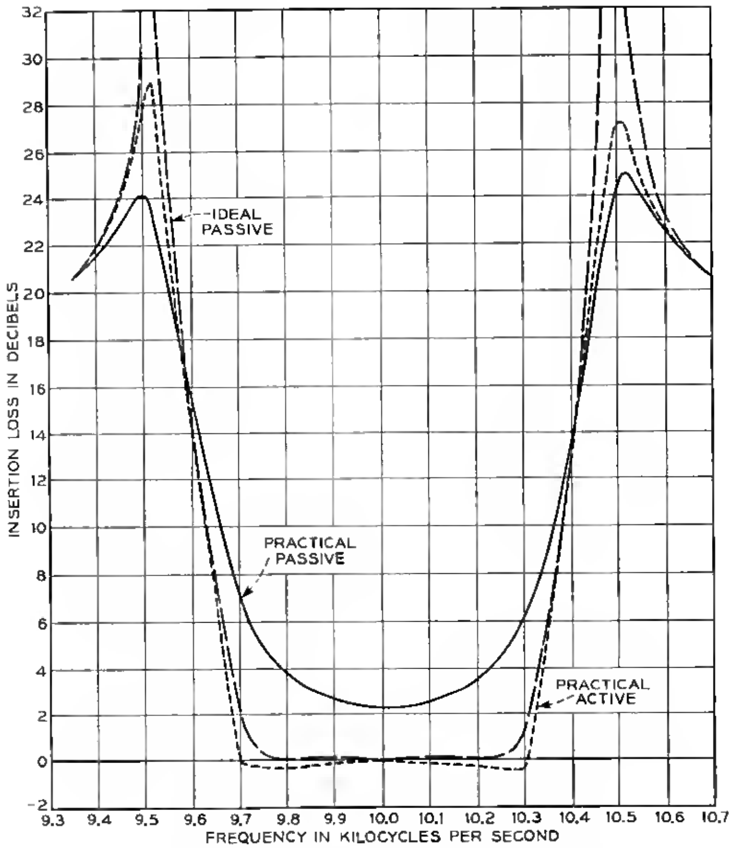


Fig. 7 — Transmission of M-derived band filters.

teristics, active filters of this kind appear to offer potential competition to the crystal channel filters used in broad band carrier systems.

Working Model

To further emphasize this fact the photograph of Fig. 9 shows a model of a composite band filter designed to transmit a 4-kc band at a midfrequency of 98 kc. This model contains seven miniature, adjustable, ferrite inductors, miniature capacitors, and two *n-p-n* junction transistors. The transmission characteristic is shown in Fig. 10. Hence in some cases by employing active circuitry it is possible to use miniature components thereby gaining at least an order of magnitude in the size and weight of structure.

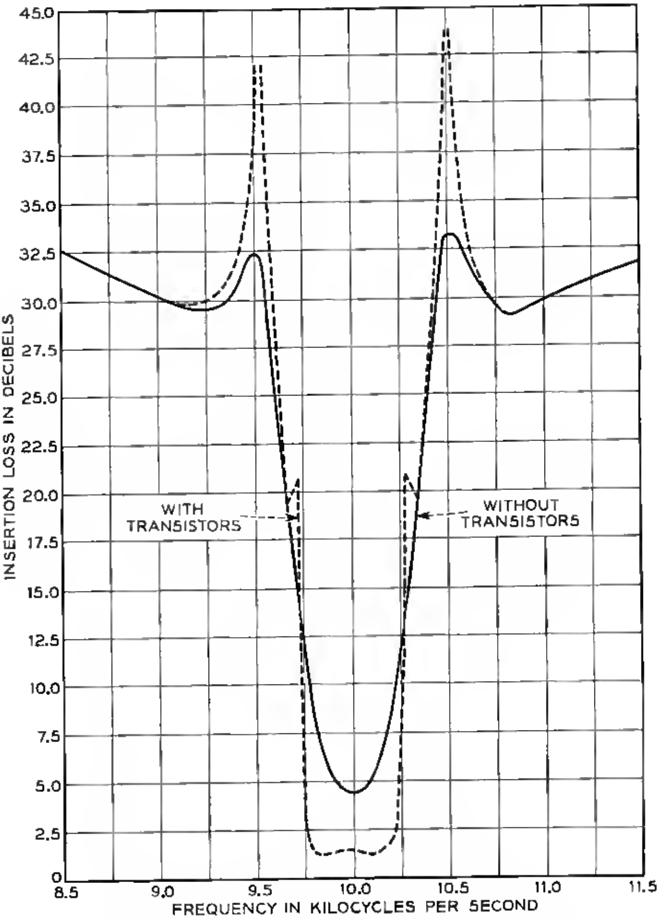


Fig. 8 — Resistance compensation of multi-section filter.

Series Negative Resistance

For satisfactory performance in many applications a series resonant circuit should approach zero impedance at the resonant frequency. To reduce the residual dissipation in an ordinary tuned circuit a series negative resistance, consisting of two transistors, can be used. This technique is illustrated in Fig. 11 which shows how a purely reactive shunt branch can be achieved in either an m-derived low pass filter or a confluent band elimination filter.

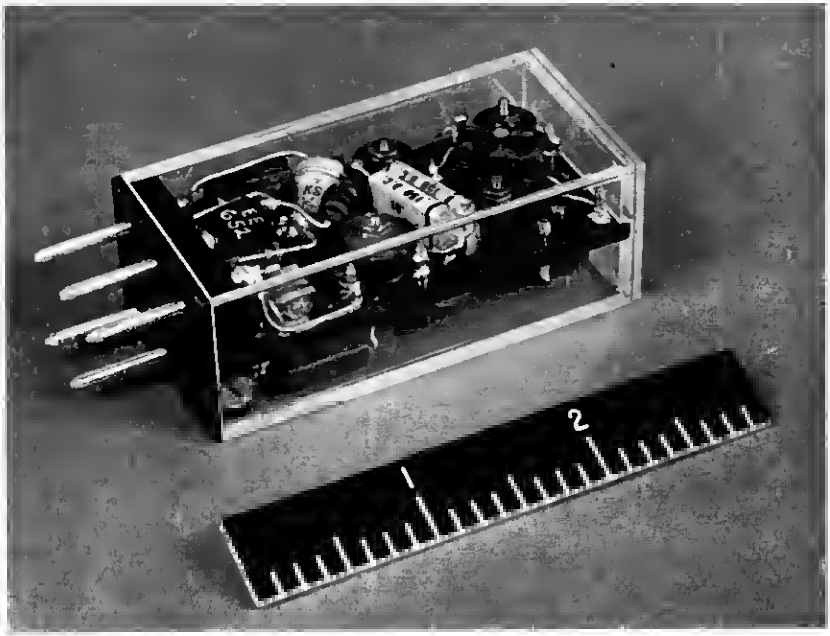


Fig. 9 — 98-kc active channel filter.

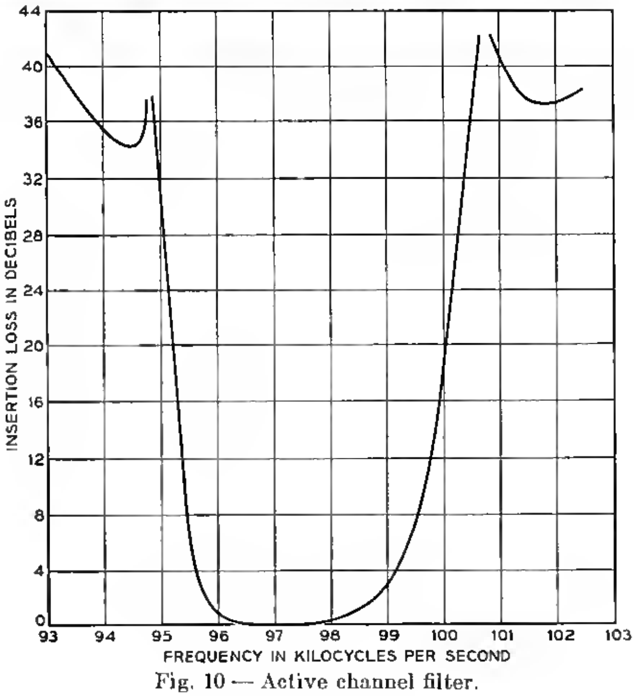


Fig. 10 — Active channel filter.

II. ELIMINATION OF INDUCTANCE

In the practical realization of frequency selective networks it is sometimes awkward, difficult, or even impossible to make effective use of coils as inductive elements. This is true, because of severe limitations on space, exacting tolerances on undesired modulation, or necessity for operation at extremely low frequencies.

It has been well known for some time that inductive elements can be eliminated without restricting the repertoire of the network designer provided he is willing to purchase this freedom by introducing active elements to supply gain.[19, 20]

It can be easily shown that the transmission through a high gain feedback amplifier is proportional to the product of the short circuit transfer admittance of the input network and the short circuit transfer impedance of the feedback network:

$$e^{-\theta} = Y_i Z_t$$

In addition it is also known from energy relations that passive networks containing only one kind of reactance cannot produce complex poles in the short circuit transfer admittance. It is instructive to consider the application of these principles to some familiar kinds of transmission networks. These networks can be logically divided into two classes: those which are primarily concerned with amplitude such as filters, and those mainly concerned with phase such as delay equalizers.

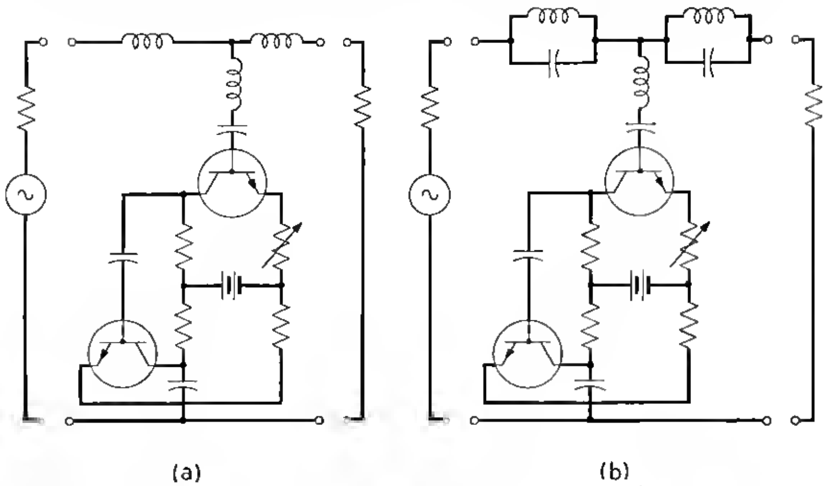


Fig. 11.— Use of series negative resistance. (a) M-derived low pass filter. (b) Band elimination filter.

Non-inductive Filters

Low Pass

Consider first an image parameter, constant-k, low pass filter which is usually built as a ladder-type structure of series inductance and shunt capacitance. A full section contains three reactive arms and produces an asymptotic loss that increases 18 db per octave.

The transmission is given by the following expression

$$e^{-\theta} = \frac{1}{(1 + \omega_0^{-1}p + \omega_0^{-2}p^2)(1 + \omega_0^{-1}p)}$$

where $\omega_0 =$ cut-off frequency in radians per second.

The function has three poles, one real and two complex conjugate. The question now arises how this function can be divided between the input and feedback networks so as to be physically realizable. Since we know that a passive R-C structure cannot have complex poles in the short circuit transfer admittance, there is no choice but to use the impedance function in the feedback circuit for this purpose. It is now found that any R-C structure which will provide the complex poles insists on providing a real zero for good measure. This unwelcome zero can be nullified by supplying its counterpart as a pole in the admittance function. The transmission is now rewritten, as follows:

$$e^{-\theta} = \left[\frac{1}{(1 + \omega_0^{-1}p)(1 + ap)}\right]\left[\frac{1 + ap}{1 + \omega_0^{-1}p + \omega_0^{-2}p^2}\right]$$

and the singularities are shown in Fig. 12(a). Since the original transmission function also requires a real pole, the admittance function must now supply two real poles. A simple ladder structure having three series resistances and two shunt capacitances meets this requirement. The complex poles cannot be supplied by a ladder structure, but require some sort of bridge such as shown in Fig. 12(a).

At low frequencies the transmission through the filter depends on the ratio of the total series input resistance to the total resistance in the bridge arm of the feedback network. Therefore any amount of flat loss or a moderate flat gain through the filter can be obtained simply by adjusting the ratio of impedance levels of the input and feedback networks.

Simulation of functions by this technique does not provide a unique solution since there is considerable freedom in choice of configuration and location of the cancelling pole and zero.

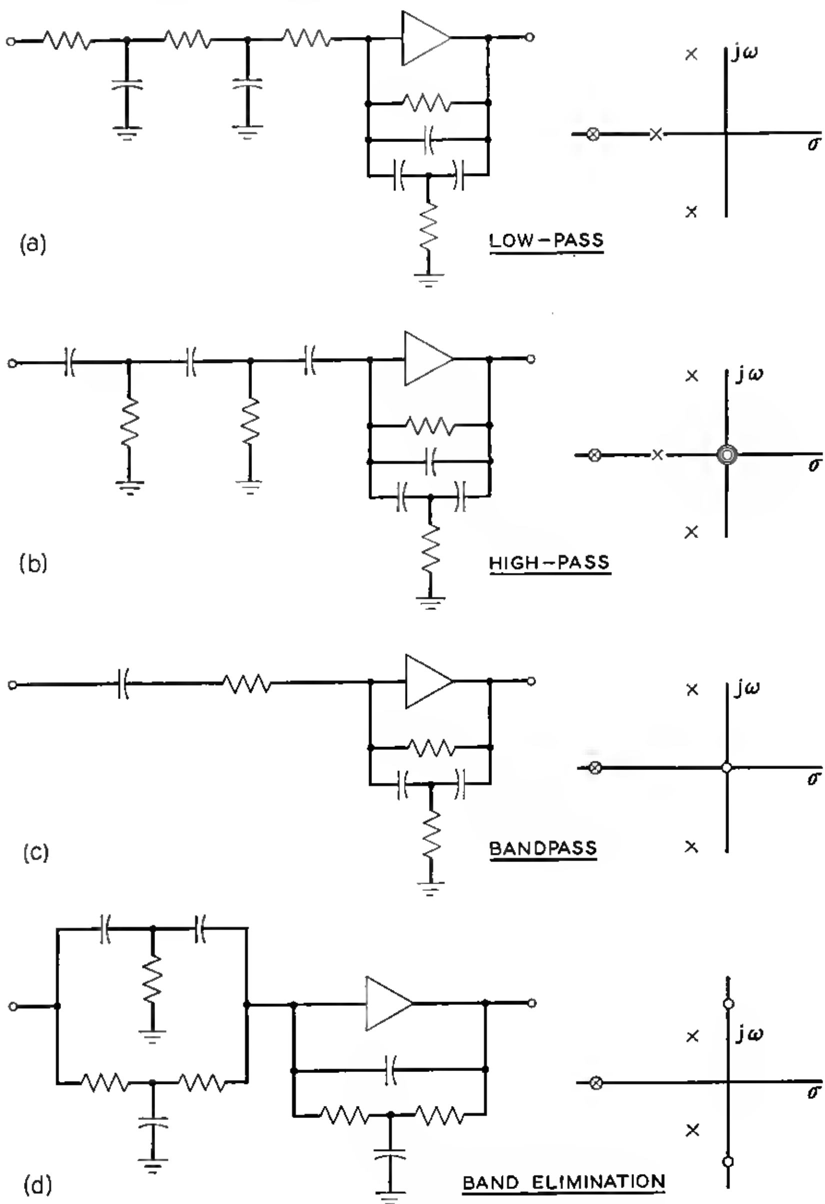


Fig. 12 — Non-inductive active filters.

High Pass

Consider next a high pass filter with cut-off at ω_0. The transmission is

$$e^{-\theta} = \frac{\omega_0^{-3}p^3}{(1 + \omega_0^{-1}p + \omega_0^{-2}p^2)(1 + \omega_0^{-1}p)}$$

$$= \left[\frac{\omega_0^{-3}p^3}{(1 + \omega_0^{-1}p)(1 + ap)}\right]\left[\frac{1 + ap}{1 + \omega_0^{-1}p + \omega_0^{-2}p^2}\right]$$

The complex plane plot in Fig. 12(b) shows exactly the same pattern of singularities as the low pass case with the addition of three zeros at the origin. To realize this function the feedback network remains unchanged, whereas the input network becomes a ladder in which the positions of the resistances and capacitances are interchanged.

Band Pass

A series resonant branch inserted in series between resistive terminations is a simple form of band pass filter having the following transmission:

$$e^{-\theta} = \frac{\omega_m^{-1}Q^{-1}p}{1 + \omega_m^{-1}Q^{-1}p + \omega_m^{-2}p^2} = \left[\frac{\omega_m^{-1}Q^{-1}p}{1 + ap}\right]\left[\frac{1 + ap}{1 + \omega_m^{-1}Q^{-1}p + \omega_m^{-2}p^2}\right]$$

where ω_m is the radian frequency of the peak and Q is a measure of the sharpness of the peak.

The singularities shown in Fig. 12(c) consist of a zero at the origin and two complex conjugate poles. Once again the complex poles are obtained by a bridge circuit in the feedback path. The usual penalty is incurred by the appearance of a real zero which must be cancelled by a real pole. Therefore the admittance function must supply a zero at the origin and one real pole. This is done by a series combination of resistance and capacitance in the input circuit.

Band Elimination

A parallel resonant branch inserted in series between resistive terminations is a simple form of band elimination filter having the following transmission:

$$e^{-\theta} = \frac{1 + \omega_m^{-2}p^2}{1 + \omega_m^{-1}Qp + \omega_m^{-2}p^2} = \left[\frac{1}{1 + ap} + \frac{\omega_m^{-2}p^2}{1 + ap}\right]\left[\frac{1 + ap}{1 + \omega_m^{-1}Qp + \omega_m^{-2}p^2}\right]$$

The singularities consist of two conjugate zeros on the real frequency axis and two complex conjugate poles. A bridge circuit in the feedback

path supplies the complex conjugate poles and a parasitic real zero, while a parallel tee in the input path provides the conjugate zeros and a real pole.

It has been found that the experimental performance of the various non-inductive filters described can be predicted with precision from the theory.

Non-inductive Phase Sections

Non-minimum phase networks are used extensively to provide a specified variation in phase with frequency without introducing any change in attenuation. Such all-pass networks consisting only of reactive elements are usually designed as lattices or bridged tee sections. It is theoretically possible and practically desirable to represent any complex all-pass structure by tandem arrangements of two basic all-pass sections called first degree and second degree. The first degree structure pro-

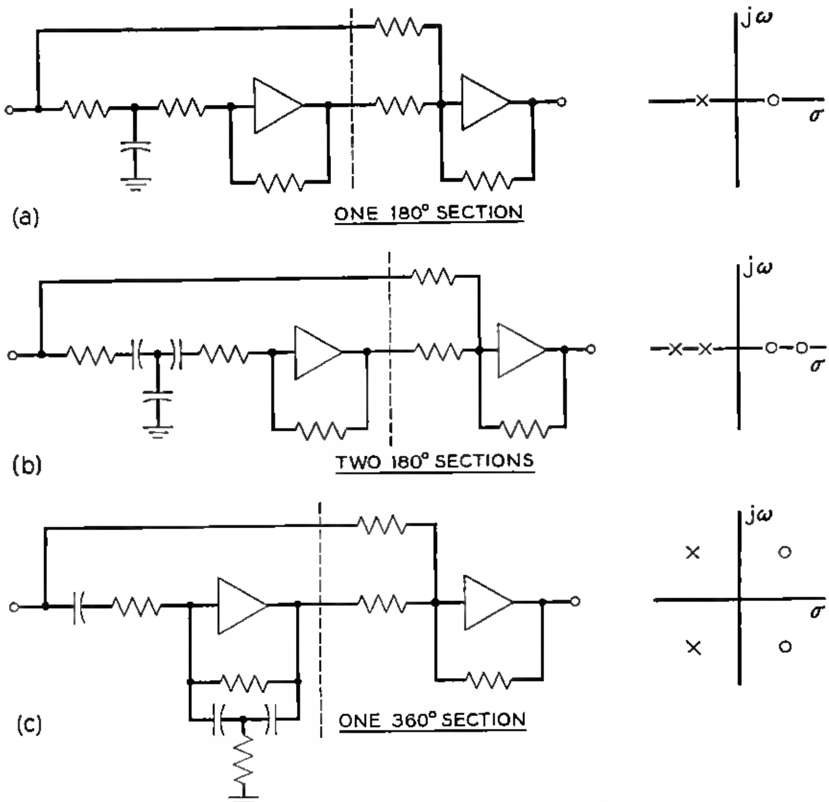


Fig. 13 — Non-inductive active phase sections.

vides a total change in phase of 180° and is characterized by a single pole-zero pair symmetrically located on the σ axis as shown in Fig. 13(a). Only one parameter, the distance from the origin can be chosen. The second degree structure provides a maximum phase shift of 360° and is characterized by two conjugate poles in the left half plane and two symmetrically located zeros in the right half plane shown in Fig. 13(c). The two parameters which can be selected are the rectangular coordinates of one singularity.

It has been suggested that these functions can be realized without benefit of inductance. Here again numerous arrangements are possible, but only a few examples will be given. The basic operation is to perform one division on the original transmission function resulting in a quotient of unity and a fractional remainder of opposite sign. The fractional remainder is then synthesized by a RC network in conjunction with an amplifier.

Single 180° Section

The transmission of a single 180° section is

$$e^{-\theta} = \frac{1 - \omega_0^{-1}p}{1 + \omega_0^{-1}p} = \frac{2\omega_0}{\omega_0 + p} - 1$$

In this case the fractional remainder consists of only one real pole which is realized by the R-C structure shown in Fig. 13(a).

Two 180° Sections

The overall transmission of two 180° sections in tandem is the product of each transmission

$$e^{-\theta} = -\left[\frac{1 - \omega_1^{-1}p}{1 + \omega_1^{-1}p}\right]\left[\frac{1 - \omega_2^{-1}p}{1 + \omega_2^{-1}p}\right] = \frac{2(\omega_1^{-1} + \omega_2^{-1})p}{(1 + \omega_1^{-1}p)(1 + \omega_2^{-1}p)} - 1$$

In this case the fractional remainder consists of one real zero and two real poles which are realized by the R-C structure shown in Fig. 13(b).

Single 360° Section

By far the most common phase corrector is the 360° section whose transmission is

$$e^{-\theta} = -\left[\frac{1 - Q^{-1}\omega_m^{-1}p + \omega_m^{-2}p^2}{1 + Q^{-1}\omega_m^{-1}p + \omega_m^{-2}p^2}\right]$$
$$= 2\left[\frac{Q^{-1}\omega_m^{-1}p}{1 + ap}\right]\left[\frac{1 + ap}{1 + Q^{-1}\omega_m^{-1}p + \omega_m^{-2}p^2}\right] - 1$$

In this case the fractional remainder consists of a zero at the origin and two conjugate complex poles which are realized by the R-C structure shown in Fig. 13(c).

III. PRODUCTION OF DELAY

It has also been proposed that a two terminal active delay equalizer can be constructed with the help of a negative resistance. As shown in Fig. 14 a two terminal network Z is connected between a resistive source and load, each of magnitude, one quarter R_0. The network Z consists of a parallel combination of a reactive network jX and a negative resistance $(-R_0)$. The transmission through Z is

$$e^{-\theta} = \frac{\frac{R_0}{2}}{\frac{R_0}{2} + Z} = \frac{\frac{R_0}{2}}{\frac{R_0}{2} + \frac{jR_0X}{R_0 - jX}} = \frac{R_0 - jX}{R_0 + jX}$$

This is the desired function, because the amplitude of the transmission

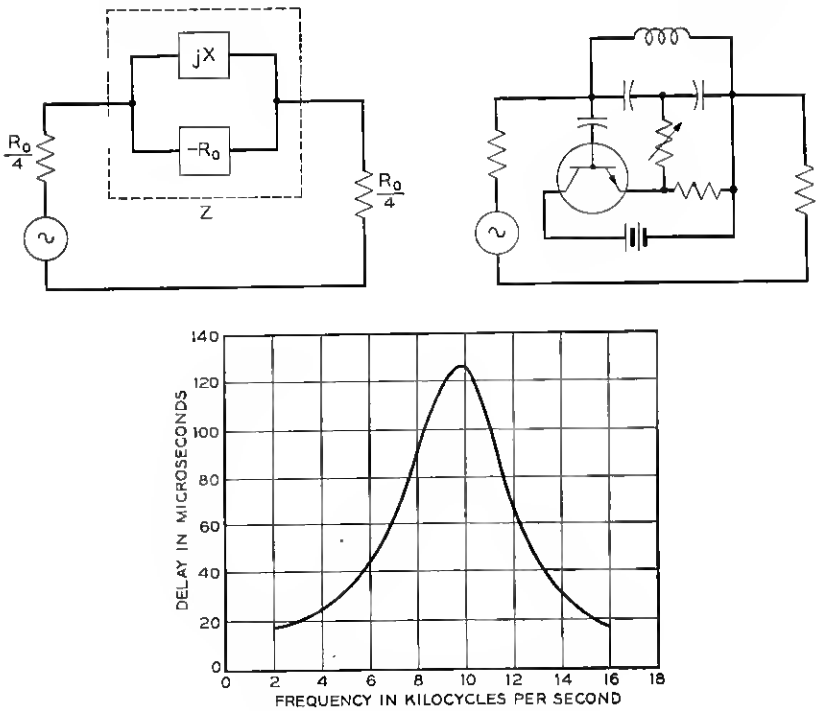


Fig. 14 — Active all-pass section.

is unity regardless of the size of X, and the phase and resultant delay are frequency dependent, because X is a function of frequency. It is theoretically possible to produce the most complicated delay equalizer characteristic by this method provided the negative resistance remains constant over the desired frequency band. As examples only a single 180° section and a single 360° section will be considered.

A 360° section results when the reactance is a single antiresonance given by

$$X = \frac{\omega L}{1 - \omega^2 LC}$$

the transmission is

$$e^{-\theta} = \frac{(p - k)^2 + \omega_m^2}{(p + k)^2 + \omega_m^2} = \frac{1 - Q^{-1}\omega_m^{-1}p + \omega_m^{-2}p^2}{1 + Q^{-1}\omega_m^{-1}p + \omega_m^{-2}p^2}$$

where

$$k = \frac{Q}{2}\omega_m \qquad \text{and} \qquad Q = (\omega_m R_0 C)^{-1}$$

In this case there are two degrees of freedom, namely, the width of the delay characteristic and the location of the peak frequency.

The circuit is shown on Fig. 14, where the transistor supplies the negative resistance, the magnitude of which is controlled by the adjustable resistance. A typical delay characteristic is also shown on Fig. 14.

A single 180° section can be obtained by simply omitting the coil in the above circuit. This is equivalent to letting $X = -(\omega C)^{-1}$ so that

$$e^{-\theta} = \frac{p - \omega_0}{p + \omega_0}$$

where $\omega_0 = (R_0 C)^{-1}$

IV. IMPEDANCE INVERSION

Two networks are said to be inverse if the product of their impedance functions is a constant. Given a network of passive elements, there are standard topological methods for finding its structural inverse if it exists. Another method is to use an active circuit in conjunction with the given impedance so that the combination offers an impedance inverse to that of the original impedance. This is a special case of modifying an impedance by feedback.[21] By means of such methods passive circuit elements can be made to appear electrically much larger or much smaller

than they really are. For example, as shown in Fig. 15(a), by using an active element a parallel combination of convenient elements such as a 0.1 henry inductance and a 10,000 $\mu\mu f$ capacitance can be made to look like the series combination in Fig. 15(b) of difficult or sensitive elements like a 100 henry inductance and a 10 $\mu\mu f$ capacitance.

This transformation can be made with the transistor circuit of Fig. 15(c) which is also drawn as the equivalent circuit of Fig. 15(d).

In the circuit of Fig. 15(c) the resistor R_A is adjusted so that the

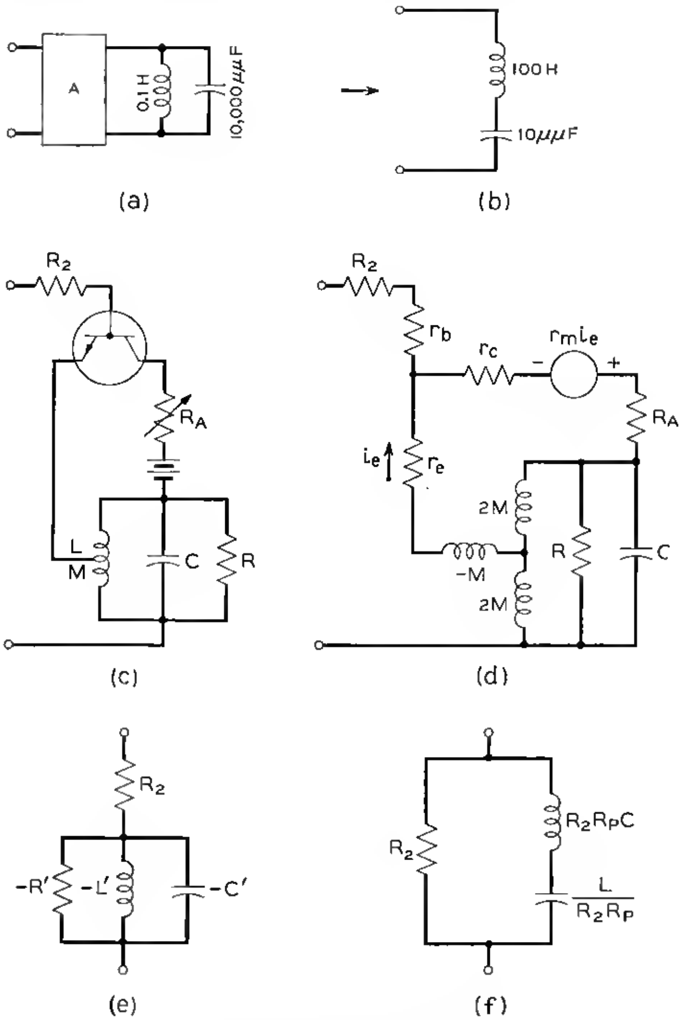


Fig. 15— Impedance inversion.

parameters of the transistor and the associated external circuit will satisfy the following equation

$$\frac{r_c}{2r_0} = \frac{2R_2}{R_P} + 1$$

To eliminate non-essentials it will be assumed that r_b and r_e are negligibly small, and $R_a \ll r_c$. Then by a straightforward, but lengthy, analysis the driving point impedance is found to be

$$Z = R_2 - \frac{pL'}{p^2L'C' + p\dfrac{L'}{R'} + 1}$$

where $L' = \lambda L$ $\qquad R_P = \dfrac{4r_0R}{R + 4r_0}$

$$R' = \lambda R_P \qquad \lambda = \frac{r_c - 2r_0}{4r_0}$$

$$C' = \frac{C}{\lambda} \qquad r_0 = r_c - r_m$$

The circuit representing this impedance is shown in Fig. 15(e). Since negative elements are not convenient a final transformation is made to the circuit shown in Fig. 15(f).

CONCLUSION

The distinctive properties of the transistor suggest careful consideration of a philosophy which regards the transistor as a circuit element to be introduced at strategic points within a network. Initial work indicates that the judicious interspersion of transistors in a transmission network makes possible performance otherwise unobtainable or uneconomical. This paper has presented examples of how transistors may be used to reduce dissipation, to eliminate inductance, to produce delay, and to invert impedance. Undoubtedly this is only the beginning of exploration which should extend the horizons of network design.

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